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Phenomenology of single transverse spin asymmetries*

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Abstract

A unified and consistent phenomenological approach to single transverse spin asymmetries in the framework of perturbative QCD, with the inclusion of a new class of spin and $\boldsymbol{k}_\perp$ dependent distribution and fragmentation functions, is presented. As an example, results on $A_N(p^\uparrow p \to \pi X)$ and $P_\Lambda(pp \to \Lambda^\uparrow X)$ are shown.

Introduction

Perturbative QCD with its factorization theorems has been in the last decades the main tool to study hard hadronic processes. An important feature of pQCD factorization theorems is the use of *collinear partonic configurations*, where collinear means that the transverse momentum ($\boldsymbol{k}_\perp$) dependence of the parton relative to the parent hadron or of the hadron relative to the fragmenting parton is integrated out. Even if this well-established approach is able to explain many properties of hard processes, a class of phenomena involving polarization degrees of freedom, like single transverse spin asymmetries, seem to be out of this understanding. Indeed according to collinear factorization theorems the asymmetry for the process $AB \to CX$ with A or C transversely polarized (relative to the scattering plane), defined as

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \tag{1}$$

($d\sigma^\uparrow \equiv E_C d^3\sigma^\uparrow/d^3p_C$ with $A^\uparrow$ or $C^\uparrow$), is almost vanishing. This comes unavoidably from the fact that the asymmetry at the partonic level is negligible. On the other hand a lot of data indicate values for such asymmetries of the order of 30-40% in size

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in some kinematical regions. In particular we refer to the observed large asymmetries in inclusive pion production in $p^\uparrow p \to \pi X$ [1] and to transverse Λ polarization in $pp \to \Lambda^\uparrow X$ [2].

In the last years a new extended formalism based on pQCD factorization theorems, including spin and intrinsic $\boldsymbol{k}_\perp$ effects, has been formulated [3]−[10], and some promising phenomenological applications have been performed. In this contribution we will summarize the main features of this approach which involves a new class of partonic distribution and fragmentation functions. We will also present and discuss the main results obtained in the understanding of such single spin asymmetries (SSA) and how we can get reasonable information on these new distribution functions by fits on available data.

Formalism

The original suggestion that intrinsic $\boldsymbol{k}_\perp$ of the quarks in the distribution functions might give origin to single spin asymmetries was first made by Sivers [3]. A similar suggestion, this time for the transverse momentum of the observed hadron relative to the fragmenting quark was later formulated by Collins [7].

More generally we can define a new class of non perturbative functions and generalize the usual factorization theorem for the analysis of $AB \to CX$ processes, at large energies and moderately large p_T, with the inclusion of spin and intrinsic (partonic) transverse momentum effects. These new, twist-two, spin and $\boldsymbol{k}_\perp$ dependent partonic distribution/fragmentation functions originate from soft, non-perturbative dynamics, which induces correlations between the intrinsic transverse momentum of, e.g., an unpolarized parton(hadron) inside (produced in the fragmentation of) a transversely polarized hadron(parton); this in turn results in an azimuthal asymmetry for the $\boldsymbol{k}_\perp$ dependence of the parton(hadron) probability distribution. The same is valid when the transversely polarized particle is the final parton(hadron).

In the sequel we will refer to the parton as a quark (q) and to the incoming hadron as a proton (p). For spin one-half hadrons we have

$$\Delta^N f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{q/p^\downarrow}(x, \boldsymbol{k}_\perp) \quad \text{[Sivers(90)]} \tag{2}$$

$$\Delta^N f_{q^\uparrow/p}(x, \boldsymbol{k}_\perp) = \hat{f}_{q^\uparrow/p}(x, \boldsymbol{k}_\perp) - \hat{f}_{q^\downarrow/p}(x, \boldsymbol{k}_\perp) \tag{3}$$

$$\Delta^N D_{h^\uparrow/q}(z, \boldsymbol{k}_\perp) = \hat{D}_{h^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{h^\downarrow/q}(z, \boldsymbol{k}_\perp) \tag{4}$$

$$\Delta^N D_{h/q^\uparrow}(z, \boldsymbol{k}_\perp) = \hat{D}_{h/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{h/q^\downarrow}(z, \boldsymbol{k}_\perp) \ \text{[Collins(93)]}\,. \tag{5}$$

Notice that by rotational invariance $\hat{f}_{q/p^\downarrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{q/p^\uparrow}(x, -\boldsymbol{k}_\perp)$ and so on.
The Sivers function, Eq. (2), is the difference between the number density $\hat{f}_{q/p^\uparrow}(x, \boldsymbol{k}_\perp)$ and $\hat{f}_{q/p^\downarrow}(x, \boldsymbol{k}_\perp)$ of quarks q, with all possible polarizations, longitudinal momentum fraction x and intrinsic transverse momentum $\boldsymbol{k}_\perp$, inside a transversely polarized proton with spin $\uparrow$ or $\downarrow$. The other functions have similar and self-explanatory

meaning. An analogous set of spin and $\boldsymbol{k}_\perp$ dependent functions is given, following the order in Eqq. (2)-(5), by $f_{1T}^\perp$, $h_1^\perp$, $D_{1T}^\perp$ and $H_1^\perp$ [9, 5, 6]. All these functions are $\boldsymbol{k}_\perp$-odd (they vanish when $\boldsymbol{k}_\perp \to 0$) and by parity invariance they have to vanish when the hadron/quark transverse spin has no component perpendicular to $\boldsymbol{k}_\perp$, so that for instance

$$\Delta^N f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) \sim k_\perp \sin\alpha \tag{6}$$

where α is the angle between $\boldsymbol{k}_\perp$ and the $\uparrow$ direction.

These functions are also T-odd, that is they would be zero due to time reversal invariance. The appearance of these distribution/fragmentation functions therefore requires that some soft initial/final state interactions are at work. However, initial state interactions might pose severe problems because they could spoil the factorization itself. The same is not true for final state interactions that can be at work among the hadron and the remnants of the fragmenting quark. Another property of these functions is their chirality: the two functions in Eq. (3) and Eq. (5) (Collins function) are chiral-odd, that is they couple quarks with left- and right-handed chiralities. This implies, since pQCD interactions conserve chirality, that they must appear together with another chiral-odd function (see below) or accompanied by a mass term. The two functions in Eq. (2) (Sivers function) and Eq. (4) are instead chiral-even and appear together with unpolarized distribution/fragmentation functions.

In this formalism, the asymmetry for the process $p^\uparrow p \to \pi X$ at leading twist and leading order in $\boldsymbol{k}_\perp$ can be expressed as

$$\begin{aligned}
2d\sigma^{\rm unp} A_N = & \sum_{abcd} \int \frac{dx_a dx_b}{\pi z} \qquad (7)\\
& \times \Big\{ \int d^2\boldsymbol{k}_{\perp a}\, \Delta^N f_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a})\, f_{b/p}(x_b)\, \frac{d\hat\sigma}{d\hat t}(x_a, x_b; \boldsymbol{k}_{\perp a})\, D_{\pi/c}(z) \\
& + \int d^2\boldsymbol{k}_{\perp c}\, \Delta_T f_{a/p}(x_a)\, f_{b/p}(x_b)\, \Delta_{NN}^{(ac)}\hat\sigma(x_a, x_b; \boldsymbol{k}_{\perp c})\, \Delta^N D_{\pi/c^\uparrow}(z, \boldsymbol{k}_{\perp c}) \\
& + \int d^2\boldsymbol{k}_{\perp b}\, \Delta_T f_{a/p}(x_a)\, \Delta^N f_{b^\uparrow/p}(x_b, \boldsymbol{k}_{\perp b})\, \Delta_{NN}^{\prime(ab)}\hat\sigma(x_a, x_b; \boldsymbol{k}_{\perp b})\, D_{\pi/c}(z) \Big\}
\end{aligned}$$

where the elementary partonic interactions are

$$\Delta_{NN}^{(ac)}\hat\sigma \equiv \frac{d\hat\sigma^{a^\uparrow b \to c^\uparrow d}}{d\hat t} - \frac{d\hat\sigma^{a^\uparrow b \to c^\downarrow d}}{d\hat t} \qquad \Delta_{NN}^{\prime(ab)}\hat\sigma \equiv \frac{d\hat\sigma^{a^\uparrow b^\uparrow \to cd}}{d\hat t} - \frac{d\hat\sigma^{a^\uparrow b^\downarrow \to cd}}{d\hat t} \tag{8}$$

and the chiral-odd transversity distribution (known also as h_1, or δq) is

$$\Delta_T f_{q/p}(x) = f_{q^\uparrow/p^\uparrow}(x) - f_{q^\downarrow/p^\uparrow}(x)\,. \tag{9}$$

The second line in Eq. (7) corresponds to the so-called Sivers effect, the third to Collins effect and the last one to the mechanism proposed by Boer [6].

Analogously for the transverse Λ polarization in $pp \to \Lambda^\uparrow X$ we have

$$d\sigma^{\rm unp} P_\Lambda = \sum_{abcd} \int \frac{dx_a dx_b}{\pi z} \tag{10}$$

$$
\begin{aligned}
&\times\Big\{ \int d^2\boldsymbol{k}_{\perp c}\, f_{a/p}(x_a)\, f_{b/p}(x_b)\, \frac{d\hat\sigma}{d\hat t}(x_a, x_b; \boldsymbol{k}_{\perp c})\, \Delta^N D_{\Lambda^\uparrow/c}(z, \boldsymbol{k}_{\perp c}) \\
&+ \int d^2\boldsymbol{k}_{\perp a}\, \Delta^N f_{a^\uparrow/p}(x_a, \boldsymbol{k}_{\perp a})\, f_{b/p}(x_b)\, \Delta^{(ac)}_{NN}\hat\sigma(x_a, x_b; \boldsymbol{k}_{\perp a})\, \Delta_T D_{\Lambda/c}(z) \\
&+ \int d^2\boldsymbol{k}_{\perp b}\, f_{a/p}(x_a)\, \Delta^N f_{b^\uparrow/p}(x_b, \boldsymbol{k}_{\perp b})\, \Delta^{(bc)}_{NN}\hat\sigma(x_a, x_b; \boldsymbol{k}_{\perp b})\, \Delta_T D_{\Lambda/c}(z) \Big\} \, .
\end{aligned}
$$

Here $\Delta_T D_{\Lambda/c}(z)$ is the analogous of $\Delta_T f_{q/p}(x)$ (see Eq. (9)).

Phenomenology

Before entering into details of fitting procedures and parameterizations of the relevant T-odd functions, we try to give a reasonable, even if qualitative, explanation of how the intrinsic $\boldsymbol{k}_\perp$ can play a crucial role in the observed spin hadronic asymmetries. We will refer to $p^\uparrow p \to \pi X$ (see Eq. (7)).

Let us consider Sivers effect alone (similar reasonings can be done for the other contributions), omitting all the unnecessary variable dependences and non relevant factors. From Eq. (6) we see that $|\Delta^N f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp)|$ reaches its maximum value at $\alpha = \pm\pi/2$. If we fix the scattering plane as the $x - z$ plane with the incoming polarized proton moving along $+\hat{\boldsymbol{z}}$, its $\uparrow$ transverse polarization is along $+\hat{\boldsymbol{y}}$. This means that $\alpha = \pm\pi/2$ corresponds to $\boldsymbol{k}_\perp$ along $\pm\hat{\boldsymbol{x}}$ (see Fig. 1). We therefore expect that at large, positive x_F ($\gtrsim 0.3$) the dominant contribution to the $\uparrow$ ($\downarrow$) polarized cross sections, assuming that only valence partons are relevant ($x_a > x_F$), is given by

$$
\begin{aligned}
d\sigma^\uparrow &\sim \sum_{q=u,d} [\hat f_{q/p^\uparrow}(+k_\perp)\, d\hat\sigma(+k_\perp) + \hat f_{q/p^\uparrow}(-k_\perp)\, d\hat\sigma(-k_\perp)] D_{\pi/q} \\
&= \sum_{q=u,d} [\hat f_{q/p^\uparrow}(+k_\perp)\, d\hat\sigma(+k_\perp) + \hat f_{q/p^\downarrow}(+k_\perp)\, d\hat\sigma(-k_\perp)] D_{\pi/q} \qquad (11) \\
d\sigma^\downarrow &\sim \sum_{q=u,d} [\hat f_{q/p^\downarrow}(+k_\perp)\, d\hat\sigma(+k_\perp) + \hat f_{q/p^\downarrow}(-k_\perp)\, d\hat\sigma(-k_\perp)] D_{\pi/q} \\
&= \sum_{q=u,d} [\hat f_{q/p^\downarrow}(+k_\perp)\, d\hat\sigma(+k_\perp) + \hat f_{q/p^\uparrow}(+k_\perp)\, d\hat\sigma(-k_\perp)] D_{\pi/q} \, . \qquad (12)
\end{aligned}
$$

Collecting together the last expressions we have for π^+ and π^- production

$$
\begin{aligned}
A_N(\pi^+) &\sim [\hat f_{u/p^\uparrow}(+k_\perp) - \hat f_{u/p^\downarrow}(+k_\perp)][d\hat\sigma(+k_\perp) - d\hat\sigma(-k_\perp)]\, D_{\pi^+/u} \qquad (13) \\
A_N(\pi^-) &\sim [\hat f_{d/p^\uparrow}(+k_\perp) - \hat f_{d/p^\downarrow}(+k_\perp)][d\hat\sigma(+k_\perp) - d\hat\sigma(-k_\perp)]\, D_{\pi^-/d} \, . \qquad (14)
\end{aligned}
$$

For the unpolarized partonic cross section, $d\hat\sigma(+k_\perp) > d\hat\sigma(-k_\perp)$ (see Fig. 1, and notice that $\theta_+ < \theta_-$), therefore in order to have $A_N(\pi^+) > 0$ and $A_N(\pi^-) < 0$ at $x_F > 0$ (see Fig. 2), we expect

$$
\begin{aligned}
\hat f_{u/p^\uparrow}(+k_\perp) > \hat f_{u/p^\downarrow}(+k_\perp) &\implies \Delta^N f_{u/p^\uparrow}(+k_\perp) > 0 \qquad (15) \\
\hat f_{d/p^\uparrow}(+k_\perp) < \hat f_{d/p^\downarrow}(+k_\perp) &\implies \Delta^N f_{d/p^\uparrow}(+k_\perp) < 0 \, . \qquad (16)
\end{aligned}
$$

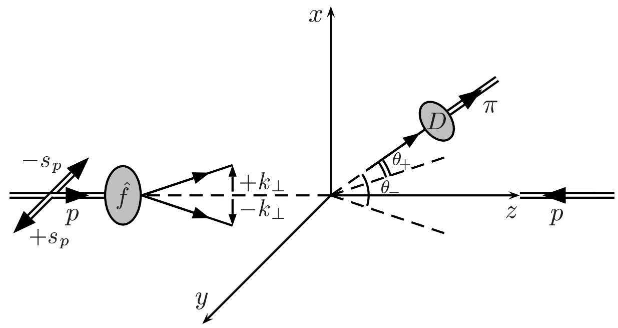

Figure 1: Pictorial representation of Sivers effect. See text for more details.

All this can help also to understand why $A_N(\pi^0)$ is not zero. In fact in this case, both up and down quarks enter, but since in the proton there are more up than down quarks we expect just a partial cancellation and still $A_N(\pi^0) > 0$. Two points are crucial here: *i*) $k_\perp$ dependence in $d\hat{\sigma}$ ($k_\perp = 0 \Longrightarrow A_N = 0$); *ii*) spin and $k_\perp$ correlations in $\Delta^N f_{q/p^\uparrow}$, which induce an azimuthal asymmetry in the partonic probability distributions ($\Longrightarrow \Delta^N f_{q/p^\uparrow} \neq 0$).

Single spin asymmetry in $p^\uparrow p \to \pi X$

A phenomenological study for this asymmetry has been carried out in a series of papers [4, 8]. For $x_F > 0$ and keeping valence contributions one can expect that the last term in Eq. (7) is not important; on the other hand in principle both Sivers and Collins effects can play a role simultaneously. However the phenomenological studies presently available take into account only one of these contributions at a time.

A fit to pion data [1] assuming Sivers effect alone and with simple parameterizations of $\Delta^N f_{q/p^\uparrow}(x, k_\perp^0)$ in the form $N_q x^{a_q}(1-x)^{b_q}$, where $k_\perp^0$ is some average value of $\boldsymbol{k}_\perp$ (see Ref. [4] for more details) gives reasonable results. The corresponding Sivers functions are acceptable, in particular they satisfy the positivity condition $|\Delta^N f_{q/p^\uparrow}| \leq 2 f_{q/p}$ with opposite sign for up and down contributions, as expected from transverse momentum conservation (see also comments above).

Analogously it has been shown that Collins effect alone could explain the data: a similar fitting and parameterization procedure has been adopted in this case [8] and the quality of the fit (see Fig. 2) is comparable to that obtained using only Sivers effect. However some comments are in order. The resulting Collins function has to saturate at large z the positivity constraint $|\Delta^N D_{\pi/q^\uparrow}| \leq 2D_{\pi/q}$. The fitted transversity distribution $\Delta_T f_{q/p}(x)$ violates the Soffer's bound. Another fit [11] which preserves this bound gives a Collins function that almost ($\sim 90\%$) saturates

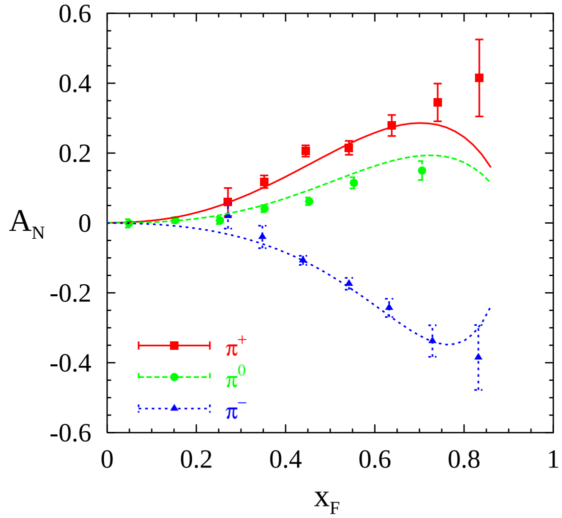

Figure 2: Best fit to A_N in $p^\uparrow p \to \pi^{\pm,0} X$ as a function of x_F at $\sqrt{s} = 20$ GeV and $p_T = 1.5$ GeV/c (Collins effect alone). Data are from Ref. [1].

over all z values and a very small transversity distribution.

Recently a similar study has been devoted to SSA in semi-inclusive DIS [10]. If confirmed, these data [12] indicate a large value of the Collins function, which might then play a significant role in other processes.

Transverse Λ polarization in $pp \to \Lambda^\uparrow X$

A huge amount of data on hyperon polarization in unpolarized $p-p$, $p-A$ collisions are available [2] but no convincing theoretical model [13] can explain them. The main features of these data, collected at $x_F > 0$, can be summarized as follows: the transverse (with respect to the scattering plane) Λ polarization is negative and can be as large as 30% in size. $|P_\Lambda|$ grows from zero as p_T increases, up to $p_T \sim 1$ GeV/c. At larger p_T it seems to show a plateau behaviour, up to the highest reachable p_T values. The value of $|P_\Lambda|$ in the plateau region increases almost linearly with x_F. On the contrary, $P_{\bar\Lambda}$ seems compatible with zero.

The first analysis of $P_{\Lambda,\bar\Lambda}$ within this formalism [14] has been recently carried out: in particular the role played by the *polarizing fragmentation function* $\Delta^N D_{\Lambda^\uparrow/q}$ (second line in Eq. (10)) has been investigated. Indeed the term in the last line is expected to give contributions at $x_F < 0$. Moreover there is some experimental evidence that the mechanism responsible for hyperon polarization should be in the hadronization process. This assumption can be tested looking at P_Λ in semi-inclusive DIS [15].

A fit on P_Λ and $P_{\bar\Lambda}$ for $p_T > 1$ GeV/c has been performed [14], assuming simple

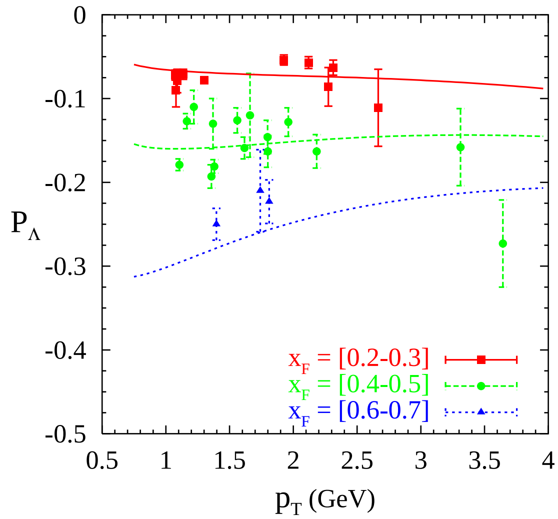

Figure 3: Best fit to P_Λ in $p\,Be \to \Lambda^\uparrow X$ as a function of p_T at $\sqrt{s} = 80$ GeV. A partial collection of data from Ref. [2] is shown.

parameterizations for $\Delta^N D_{\Lambda^\uparrow/q}$. With a reasonable set of parameters we are able to reproduce all the main features of data (see Fig. 3). In particular, it results $\Delta^N D_{\Lambda^\uparrow/u,d} < 0$, $\Delta^N D_{\Lambda^\uparrow/s} > 0$ (this can help to explain the different behaviour of P_Λ and $P_{\bar\Lambda}$) and $|\Delta^N D_{\Lambda/u,d}| < \Delta^N D_{\Lambda/s}$, independently of the set of unpolarized fragmentation functions adopted.

Conclusions

We have presented a unified and consistent formalism, derived in the framework of pQCD, by extending the usual factorization theorems and including a new class of T-odd, twist-two, spin and $\boldsymbol{k}_\perp$ dependent distribution and fragmentation functions. We have shown as it allows to describe and explain the amount of data on single transverse spin asymmetries observed in hadronic reactions at moderately large p_T. A combined theoretical and experimental analysis of several processes will allow to improve our knowledge on such phenomena and test more deeply this formalism.

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